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Estimation of Thermal Properties and Surface Heat Flux in Carbon-Carbon Composite

K. Dowding,* J. Beck,† and A. Ulbrich‡
Michigan State University, East Lansing, Michigan 48824
B. Blackwell§
Sandia National Laboratories, Albuquerque, New Mexico 87185
and
J. Hayes¶
Wright Laboratory, Wright-Patterson Air Force Base, Ohio 45433

This article discusses a laboratory method of measurement of the thermal properties of a carbon matrix carbon fiber composite material. Another important objective is the determination of the surface heat flux from measured temperatures within the body. This article is unique because the thermal properties are being estimated with known heat flux and temperature histories; then the opposite problem of recovering the heat flux history is solved using the estimated values for the thermal properties and temperature history. This calculated heat flux history can then be compared with the measured input heat flux. Results were obtained for a temperature range of 30–600°C. The thermal properties demonstrate a presumed quadratic relationship with temperature, and a good agreement between the estimated and measured heat flux histories is demonstrated.

Nomenclature

C = specific heat, J/kgC
e = temperature residual, °C
Fo = Fourier number
L = length, m
$\dot{q}''$ = heat flux, W/m^2
T = temperature, °C
$\hat{T}$ = calculated temperature, °C
t = time, s
Y = measured temperature, °C
κ = thermal conductivity, W/mC
ρ = density, kg/m^3
Ω = resistance, ohms

Subscripts
cc = carbon-carbon composite material
ins = insulation material
y = direction normal to the fiber direction

Introduction

Problem Description

THE development of composites materials is proceeding at an ever increasing rate. Advanced materials are used in automotive, airplane, and aerospace applications. The strength-to-weight ratio and survivability in harsh conditions are two main advantages of these materials. However, the emergence of these materials requires experimental techniques and solution methods to determine their thermal properties. A related problem is to find the surface heat flux from interior temperature histories when the composite bodies are heated by unknown conditions. This article is about both the estimation of thermal properties from temperature and heat flux measurements and the estimation of the surface heat flux from the same temperature measurements for a composite material.[1,2]

The investigated material is a carbon matrix-carbon fiber material made by Carbon-Carbon Advanced Technologies, Inc., of Fort Worth, Texas. There are six specimens, 7.62 cm square and 0.953 cm thick. (Testing, presented herein, was performed on two of these specimens.) The specimens are described (by the manufacturer) as CC1 2-D composite made from fiberite K-641 fully densified, SiC pack coated with sealant. Because the specimens were not flat as received, the specimens were ground to produce flat surfaces.

Motivation

Carbon matrix-carbon fiber materials have the capability of structural integrity at very high temperatures and have been considered for advanced flight vehicles. Unfortunately, the production and curing processes are not yet consistent in the production of materials that have uniform thermal properties from batch-to-batch. As a consequence, a device for measuring thermal properties of these materials is needed. This article discusses a laboratory method of measurement of the thermal properties (as a function of temperature).

To fully characterize the thermal properties (thermal conductivity and volumetric heat capacity) requires a multidimensional investigation due to the construction and type of materials in the carbon-carbon composite. This is particularly true for the case of sample-to-sample variations and the need to measure the properties on a vehicle in field conditions. The thermal conductivity varies in directions normal and parallel to the fibers. However, only one-dimensional experiments are considered in this article and the thermal conductivity normal to the fibers is determined, which is denoted κ_y and volumetric heat capacity ρC.

A unique aspect of this research is the recovery of the heat flux after the thermal properties are determined. That is, the transient measurements of heat flux and temperature are used

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*Research Assistant, Department of Mechanical Engineering. Student Member AIAA.
†Professor, Department of Mechanical Engineering.
‡Visiting Research Assistant, Department of Mechanical Engineering.
§Member Technical Staff, Thermophysics Department. Associate Fellow AIAA.
¶Aerospace Engineer, WL/FIMA.

to estimate the thermal properties; then, these thermal properties and transient temperature measurements are used to recover the measured heat flux. The later problem is the well-known inverse heat conduction problem (IHCP). This "completing the circle" of parameter estimation and then estimation of the surface heat flux has not, to our knowledge, been done before. Although new information is not gained by completing the circle, it does verify the consistency of the inverse solution methods. Furthermore, the capability of estimating the surface heat flux, after estimating the thermal properties, is not a trivial exercise given the ill-posedness of the IHCP problem. The process may provide valuable insight to the methods, particularly since actual experimental data is used.

Literature Review

The theory of the technique in this research using electric heaters for estimating the thermal properties is detailed in the book by Beck and Arnold[3] (see Chapter 7 in particular). A consequence of using electric heaters is that typically both thermal properties can be estimated simultaneously. Hence, this research uses the electric heaters.

Other methods with known heat flux and transient measurements are described.[4-7] The first of these papers[4] uses an internal heat flux transducer. The papers by Scott and Beck[5,6] relate to composite materials. The last[7] sequentially estimates properties by mathematically connecting a series of discrete experiments with various temperature ranges. Loh[8] has given results for determining two components of the thermal conductivity for a carbon-carbon composite. The papers by Garnier et al.[9] describe a method for estimating thermal properties without having the temperature sensor inside the specimen(s). This has also been attempted in the present work, but the method (at least for one-dimensional cases) is only appropriate for materials with quite low thermal conductivities because of the contact conductance. The carbon-carbon composite material does not have low values, however. Also relevant is the study of optimal experiments, which is discussed in Chapter 8 of Beck and Arnold[3] and in Taktak et al.[10]

The determination of the surface heat flux from interior temperature histories is called IHCP. Three books on this subject are written by Refs. 11–13. The paper by Beck and Osman[14] emphasizes the function specification method using a variable number of future times. In Osman and Beck[15,16] one- and two-dimensional problems associated with quenching are discussed.

Experimental Aspects

A sketch of the experimental setup used to estimate the thermal properties of the carbon-carbon material is shown in Fig. 1. It consists of two nominally identical carbon-carbon specimens (7.62 × 7.62 × 0.914 cm), and ceramic insulations (7.62 × 7.62 × 2.54 cm, Zircar Products Inc., Florida, New York), with a mica heater assembly [Thermal Circuits, Inc., Salem, Massachusetts, $\Omega(T_{\mathrm{room}}) = 33\ \Omega$] located between the identical halves. Five thermocouples (type E, 0.254-mm nom. wire dia.) are embedded on the surface of each carbon-carbon specimen at the heater/specimen interface. The thermocouples (insulation removed) are cemented into grooves (0.38 mm wide × 0.46 mm deep) that extend the length of the specimen. Two thermocouples are at each interface of the carbon-carbon specimen and the ceramic insulation. The entire setup is mounted between two 3.175-mm-thick aluminum plates that are connected with threaded rods and hold the layers firmly in place; the setup is placed in a furnace, which allows the initial temperature to be varied.

During the initial stages of this research a different heater design was used. The heater was constructed of a Kapton® material with resistance temperature sensors, which were integral in the heater assembly. This approach had the advantage of being nonintrusive. However, because high temperatures (beyond the temperature range of the Kapton, >200°C) were required and the sensitivity to the contact conductance is much greater with the sensors located in the heater assembly, the sensors were embedded in the surface of the specimen.

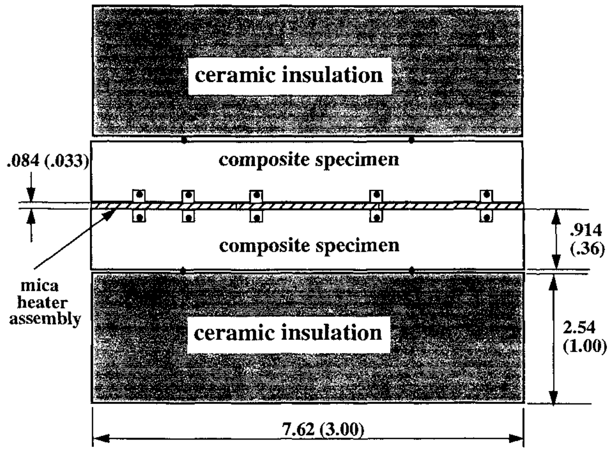

Fig. 1 Experimental setup.

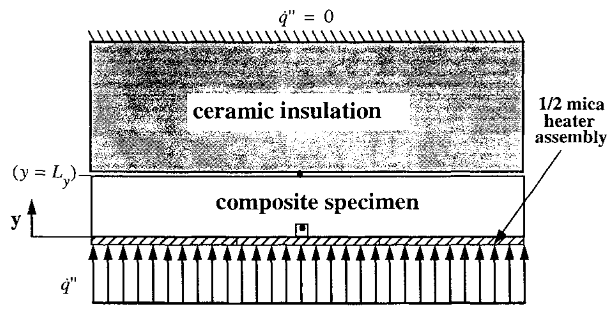

Fig. 2 Heat transfer model for one-dimensional experiment.

The experiments are conducted and processed using a 12-bit data acquisition system (National Instruments) with a 486 PC. The system provides accurate data acquisition with minimum sampling intervals in the μs range. Two data acquisition boards are linked providing 16 channels of data acquisition. The system controls and samples the power (voltage and current) delivered to the heaters and samples the thermocouple voltages. The current is acquired by measuring the voltage across a known resistance. The thermocouple raw data (voltages) are converted to temperatures and corrected to eliminate bias. The heat flux is calculated from the power measurements assuming the heating is uniform over the heating area (7.62 × 7.62 cm).

The model of the experimental setup is shown in Fig. 2. The energy to the heater is assumed to divide equally between the two halves, emanating from the midplane of the mica heater assembly ($y = -0.042$ cm). The temperatures at the two measurement locations ($y = 0$ and $y = L_y$) are the average of all corrected thermocouples at these locations. The sensors embedded in the specimen are assumed to measure the temperature at the surface of the specimen. An insulation boundary condition at the back of the specimen is chosen, instead of a temperature boundary condition (which was initially used), to minimize the sensitivity to the contact conductance at this location. Since ($\kappa_{y,cc} >> \kappa_{ins}$), small temperature gradients will exist near the specimen/insulation interface, and the nonembedded sensors measure the temperature at the backside of the specimen. A temperature boundary condition, however, has a relatively large heat flux at the boundary. Consequently, a nonembedded sensor is very sensitive to both the contact conductance and sensor location. Hence, an insulating boundary condition is used.

Including the mica heater and ceramic insulation in the model (Fig. 2) requires the properties of the mica and insulation to be known, or also estimated, to determine the properties of the carbon-carbon specimen. Neglecting the mica heater in the model is not appropriate because the contact conductance results in a large temperature drop across the heater. If the heater is neglected, the carbon-carbon properties will incorrectly reflect this effect. Also, including the heat loss to the insulation, instead of assuming a perfect insulated condition (at the specimen/insulation interface), increases the accuracy of the estimated properties for the carbon-carbon. If known properties are used for these materials (mica and insulation) several problems arise. First, the properties are not typically known very accurately. Secondly, contact conductance between the layers is typically not negligible and must be considered. Third, the ceramic insulation had to be treated to strengthen its mechanical structure. By experimentally estimating effective properties of these materials, these problems are solved.

This approach requires additional experimental work, however. The experimental conditions (length of experiment and heating duration) to determine the properties of the mica and insulation are quite different from the conditions necessary to estimate the properties of the carbon-carbon. A separate series of tests are performed without the carbon-carbon specimen to determine the properties of the insulation and mica heater assembly. These effective properties account for any contact conductance due to imperfect contact between the different material layers.

With the separate series of experiments, the effective thermal properties of the ceramic insulation and mica heater assembly were determined. However, when the setup is reconfigured to test the carbon-carbon composite, the contact conductance may vary. Therefore, for each experiment to determine the properties of the carbon-carbon composite, a short duration experiment is conducted to characterize the effective thermal properties of the mica heater and contact conductance.

For a typical set of experiments (to determine the properties of the carbon-carbon composite), the furnace is set at a given temperature and the experimental apparatus is allowed to reach a uniform temperature (usually this required 1 h). A short duration experiment (heating for 2 s with 0.01-s sampling interval) is run first, which is used to estimate the effective properties of the mica heater. After allowing the setup to again stabilize at a uniform temperature, a second experiment is run. The second experiment (heating for 20 s with 0.16-s sampling interval) is longer in duration and is used to estimate the properties of the carbon-carbon composite specimen.

Analysis Procedures

The techniques used to estimate the thermal properties and evaluate the IHCP are detailed in books by Beck and Arnold[3] and Beck et al.,[11] and in several papers mentioned above. The basic process involves minimizing a sum of squares function S between the measured and calculated temperature

$$S = \sum_{i=1}^{I} \sum_{j=1}^{J} (Y_{ij} - \hat{T}_{ij})^2 \tag{1}$$

where I is the number of measurement times, and J is the number of sensors. To determine the thermal properties this function is minimized with respect to the thermal properties. Similarly, the function is minimized with respect to the surface heat flux to solve the IHCP.

These solution procedures are implemented with two computer programs. PROP1D is used to estimate the thermal properties. The surface heat flux is determined using IHCP1D.

PROP1D provides a means to estimate thermal properties of multilayer bodies from appropriate transient temperature and heat flux measurements. Thermal conductivity and volumetric heat capacity may be determined simultaneously and for more than one material, if desired. Layers of different materials may also be lumped and effective thermal properties determined. Garnier et al.[17] performed experiments on materials with well-known published thermal properties to support the accuracy of PROP1D.

The IHCP is evaluated using the program IHCP1D. The program provides a means to determine unknown conditions at the boundary of a body from transient temperature measurements within the body. The body may be composed of different materials. The material thermal properties are required and may be functions of temperature. The algorithm uses the function-specification-method and the concept of future information to determine the unknown heat flux at the surface.[11] The number of future time steps used in the analysis is variable, which allows (with minimal error) determining heat fluxes that are both smooth and vary rapidly during an experiment.

Results and Discussion

Parameter Estimation

The determination of the effective properties of the mica heater is an integral step in determining the properties of the carbon-carbon composite. It was found that the estimation of thermal conductivity and volumetric heat capacity simultaneously is not possible, due to correlated sensitivity coefficients (see Beck and Arnold[3]). Therefore, the volumetric heat capacity of the mica heater is set at the value estimated during the experiments to determine the properties of the ceramic insulation, and the effective thermal conductivity of the mica heater is estimated. The thermal conductivity is estimated because it is most influenced by the contact conductance.

The effective thermal conductivity estimated for the mica heater assembly, assuming $\rho C = 2.0 \times 10^6$ J/m^{3}C, demonstrated no definite trend with temperature over the range of 30–295°C. The largest value is κ (30 deg) = 0.14 W/mC, and the smallest is κ (134 deg) = 0.10 W/mC; results at other temperatures are within this range. For temperatures up to 295°C the thermal conductivity of the mica heater is estimated at each temperature. After which, the magnitudes of the thermal properties were shown to minimally affect the outcome of the estimation for the carbon-carbon properties. Variation of 50% in the thermal properties used for the mica heater results in variations in the estimated properties of the carbon-carbon that are within the magnitude of the confidence intervals (discussed below). For temperatures greater than 295°C, the thermal conductivity estimate of the mica heater at 295°C is used in the analysis. The fact that it is difficult to estimate the thermal properties of the mica heater means that these properties are not significant in the estimation of the carbon-carbon properties (or model), as discussed above. This is a desirable characteristic for additional materials (i.e., mica heater and ceramic insulation) in the experimental model.

The properties determined for the carbon-carbon composite are given in Table 1. The first column identifies the experimental case, and the second column identifies the initial temperature. The third column is the root-mean-square (rms) of the analysis, which is defined as

$$\text{rms} = \left[\frac{1}{(N_t - N_p)} \sum_{i=1}^{I} \sum_{j=1}^{J} (Y_{ij} - \hat{T}_{ij})^2\right]^{0.5} \tag{2}$$

where N_t is the total number measurements (from both sensor locations), and N_p is the number of parameters being estimated. I and J are the same as defined for Eq. (1). The last two columns present the estimated properties with the associated confidence interval (calculated by PROP1D and discussed below).

Table 1 Estimated thermal properties for carbon-carbon composite material

Experimental case	Initial temperature, °C	rms, °C	$\kappa_{y,cc}$, W/mC	$(\rho C)_{cc} \times 10^{-6}$, J/m^3 C
1010#30.1	31	0.128	3.40 ± 0.05	1.42 ± 0.01
1010#41.1	42	0.114	3.48 ± 0.04	1.47 ± 0.01
1012#94.1	95	0.161	3.85 ± 0.07	1.74 ± 0.02
1012#108.1	109	0.161	3.93 ± 0.07	1.81 ± 0.02
1011#143.1	143	0.121	4.12 ± 0.07	1.90 ± 0.02
1011#159.1	159	0.089	4.22 ± 0.06	2.08 ± 0.01
1012#195.1	195	0.091	4.35 ± 0.06	2.20 ± 0.02
1013#259.1	259	0.075	4.60 ± 0.04	2.47 ± 0.02
1008#295.1	295	0.090	4.76 ± 0.05	2.52 ± 0.02
1013#304.1	304	0.103	4.74 ± 0.05	2.58 ± 0.02
1023#403.2	403	0.100	4.86 ± 0.05	2.76 ± 0.02
1023#455.2	455	0.082	4.93 ± 0.03	2.88 ± 0.02
1023#508.2	508	0.096	4.99 ± 0.05	2.97 ± 0.02
1023#571.2	571	0.278	3.93 ± 0.23	3.06 ± 0.10
1023#623.2	623	0.205	3.73 ± 0.15	3.23 ± 0.07

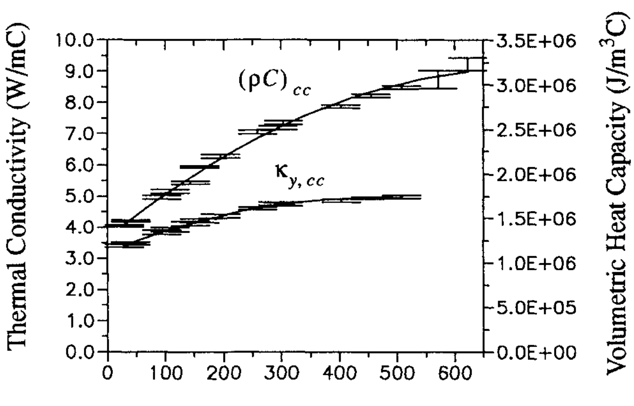

Fig. 3 Temperature dependence of thermal properties.

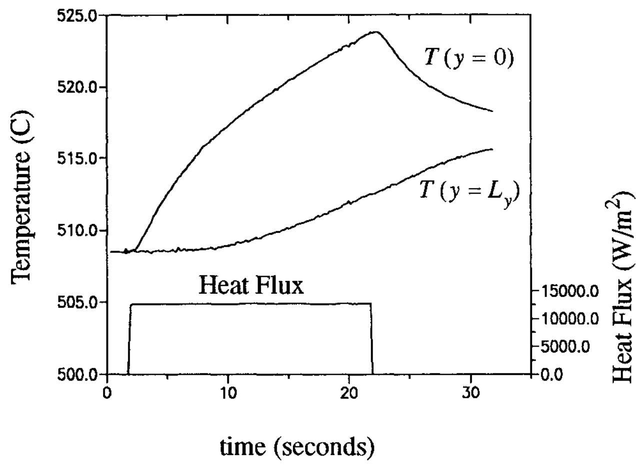

Fig. 4 Experimental data for case 1023#508.2.

Insight about the estimated properties is gained by plotting the properties as a function of temperature; Fig. 3 gives a plot of the one-dimensional properties of the carbon-carbon composite as a function of the temperature with confidence intervals. Note that the analysis assumed that the thermal properties were constant during an experiment, but varied between experiments; the initial temperature of the experiment is used to plot the properties. Using an F-test[3] it is concluded that a second-order (in temperature) model adequately represents the results. The equations determined for the properties with a least-squares fit are

$$\kappa_{y,cc} = 3.195 + 0.00756T - 0.813E\text{-}05T^2 \quad (3)$$

$$(\rho C)_{cc} = (1.280 + 0.00526T - 0.363E\text{-}05T^2) \times 10^6 \quad (4)$$

The relationships are shown in Fig. 3. The thermal conductivities determined from the last two experiments are not used in the least-squares analysis. The validity of the thermal conductivity at temperatures of 571 and 623°C is uncertain. The residuals and confidence intervals for these two cases are relatively large. When the setup was disassembled after these experiments, the outer coating of the specimen (which protects from oxidation), had separated from the inner composite material. Because the estimated thermal conductivities are lower, it is quite possible that the failure of the specimen occurred before or during these experiments. The volumetric heat capacity (at these temperatures), which would not be affected greatly by this type of failure, correlates with the rest of the results and is used in the least-squares analysis.

The details of the parameter estimation are quite similar at different initial temperatures. For this reason, and for brevity, only one case (1023#508.2), at an initial temperature of 508°C, is discussed. The experimental data for this case is presented in Fig. 4. The temperatures shown represent the average of all thermocouples at the referenced location. The results closely approximate the case of a finite slab heated with a constant heat flux at the surface and insulated at the backside. Initially, the temperature at the heated surface increases rapidly, while the backside temperature remains constant. After which, the temperatures at the heated surface and backside both increase linearly with time until the power to the heater is turned off. Then the temperatures at both surfaces tend to approach the same temperature, demonstrating that there is little heat loss to the insulation (even though it is considered in the analysis).

In addition to estimating the thermal properties, PROP1D provides some means to quantify the accuracy of the estimates. The previously discussed rms [Eq. (2)] provides an indication of how well the measured temperatures match the calculated temperatures. The magnitude of the rms can be compared to the temperature rise of the experiment, which is approximately 15°C. Except for the last two experiments, the rms is within 1% of the temperature rise. Also given with the estimates of the parameters is a confidence interval (Beck and Arnold,[3] Chapter 6). The calculation of the confidence interval has some associated assumptions. First, the model for the experiment is correct. Second, the dominant errors in the analysis are in the temperature measurements, modeled with a first-order autoregressive model,[3] and the errors are not biased.

There are other quantities that can be observed to demonstrate the accuracy of the estimated properties. These include the sequential estimates of the properties, the residuals, and the sensitivity coefficients. Each is discussed below.

The sequential estimates demonstrate how the estimated properties vary as additional measurements are considered. Figure 5 shows the sequential estimates for this case. The sequentially estimated property, at time t_i, represents the outcome if only data up to that time is used in the analysis. In other words, if the data is analyzed by adding one data pair $[Y(y = 0)$ and $Y(y = L_y)]$ at each time, it shows how the estimated properties change as one more data pair is added to the analysis. Initially, the sequential estimates vary because there is not enough information to determine the properties. However, as more data is considered, the property estimates approach constants. Meaning, if the experiment (or analysis) is ended at, e.g., 15 s, the estimated properties would not differ significantly from the properties at 30 s. In general, for a good estimation, the sequential estimates converge to a constant and are fairly steady with time. For times greater than 15 s, the sequential estimates of $\kappa_{y,cc}$ and $(\rho C)_{cc}$ vary 0.9 and 3.5%, respectively.

The residuals are related to the rms and are calculated as follows:

$$e_{ij} = Y_{ij} - \hat{T}_{ij} \tag{5}$$

They represent the difference between the measured and calculated temperature for a particular time (t_i) and sensor location (j). The rms gives an indication of the magnitude of the residuals; the signs and magnitudes of the residuals can provide considerable insight. Figure 6 presents the residuals for this case. The magnitude of the residuals is approximately 0.1, which is not unexpected considering that the rms for this experiment is 0.1. There is some correlation in the residuals; most of the residuals are positive during the heating. This outcome may signify that some inconsistency exists in the model or that a small effect was omitted. However, the magnitudes of these residuals are small, within 1% of the temperature rise during the experiment, indicating that errors in the model are minimal.

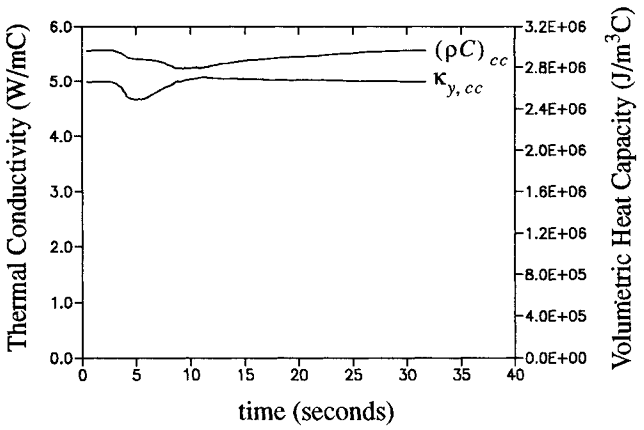

Fig. 5 Sequential parameter estimates for case 1023#508.2.

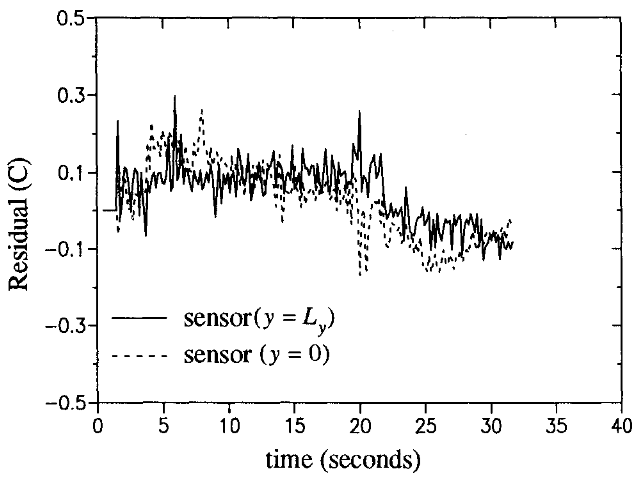

Fig. 6 Temperature residuals for case 1023#508.2.

Sensitivity coefficients are the first derivative of the temperature with respect to the properties, thermal conductivity and volumetric heat capacity. They provide indicators of how well-designed the experiment is. In general, the sensitivity coefficients are desired to be large and uncorrelated (linearly independent). A sense of the magnitude of the sensitivity coefficients is gained through normalizing the sensitivity coefficients by multiplying by the properties, resulting in units of temperature for the normalized sensitivity coefficients. A comparison is then possible with the temperature rise of the experiment. For a well-designed experiment, with boundary conditions similar to the case investigated in this study, the sum of the normalized sensitivity coefficients for the thermal conductivity and volumetric heat capacity is equal to the negative of the temperature rise. Sensitivity coefficients are useful in the design of experiments, i.e., determining the heating and experiment duration, location of sensors, heating area (for two-dimensional case), etc. A study of the sensitivity coefficients, prior to performing experiments, can lead to better experiment designs.

Figure 7 shows the sensitivity coefficients for a representative experimental case. The sensitivity to the thermal conductivity and volumetric heat capacity are shown for both sensor locations. Notice that the sensitivity coefficients are correlated (linearly dependent) for times up to 10 s for the sensor at the surface of the specimen ($y = 0$). This is similar to the situation that resulted in only being able to estimate one parameter for the effective properties of the mica heater in the analysis of the short duration experiment. In this case, however, information is available from another sensor ($y = L_y$), and at later times when the sensitivity coefficients are not correlated.

IHCP

The IHCP is difficult for two reasons, the information used to determine the heat flux at the surface is damped and delayed. Because the sensors are above the plane of the heater element (Fig. 2), the effect that the heat flux has at the sensor location is lagged in time, and the magnitude of the effect is damped (smaller). As the sensors are moved away from the surface, the temperature response at the sensor is caused by a heat flux that occurred some time earlier; this temperature response becomes smaller in magnitude. The farther away from the surface the sensor(s) is moved, the more pronounced are the lagging and damping effects.

Evaluating the surface heat flux with data from both sensor locations ($y = 0$ and $y = L_y$) is shown in Fig. 8. The agree-

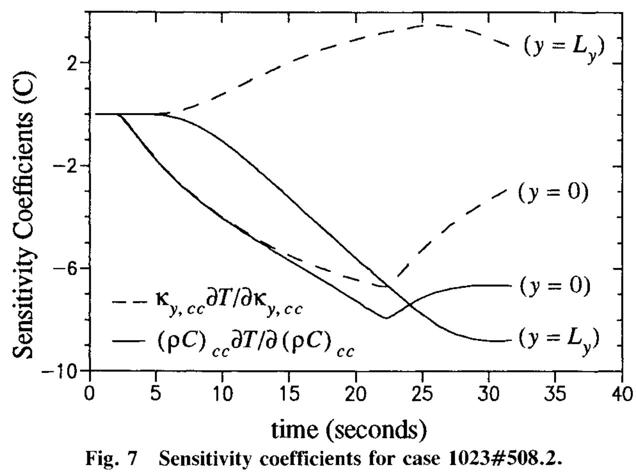

Fig. 7 Sensitivity coefficients for case 1023#508.2.

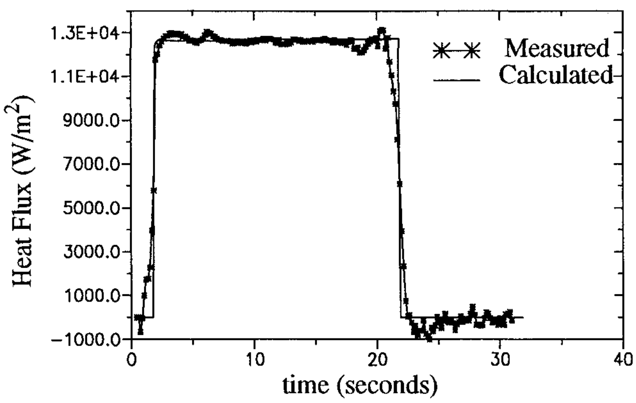

Fig. 8 Estimated surface heat flux for case 1023#508.2.

ment between the measured and estimated values is excellent; the rms is 0.170. The greatest inaccuracy occurs when the heater is powered off or on; but even there the agreement is satisfactory.

Although the IHCP is less difficult when the sensors are close to the surface (where the heat flux is applied), in reality the case presented here is more difficult than it appears. The heat flux (to estimated) is separated from the nearest sensor by one-half the thickness of the mica heater, which has a relatively low thermal conductivity and there is a contact conductance. This thin, but very low conductivity, material adds to the difficulty. To demonstrate this point, consider the Fourier number for the sensor location based on the effective properties of the mica heater and time step[11]

$$Fo = \frac{\alpha \Delta t}{L^2} = \frac{(0.1/2.0 \times 10^6)0.16}{(0.00042)^2} = 0.045 \tag{6}$$

where L is the thickness of the mica heater (or sensor depth). This is a very small Fo value, indicating a difficult IHCP. If the heater were the same material as the specimen, the equivalent sensor depth (based on the properties of the carbon-carbon composite) is

$$L = \left[\frac{(4.99/2.99 \times 10^6)0.16}{0.045}\right]^{0.5} = 0.24 \text{ cm} \tag{7}$$

or about 30% of the thickness of the specimen. The low thermal conductivity of the mica heater significantly increases the difficulty of this problem by delaying and damping the response of the heat flux at the sensor location.

The function specification method,[11] which is used to estimate the surface heat flux, uses the concept of future information. Since there is a time delay at the sensor location to a change in the surface heat flux, the data at later times ($t_0 + \Delta t$, $t_0 + 2\Delta t$, . . . , $t_0 + r\Delta t$) is used to calculate the surface heat flux at time t_0, where r is the number of future time steps. This procedure also has the effect of stabilizing the solution. The number of future time steps used is 12 during the time period $2.0 < t < 20$ s, and 6 otherwise. Although the form of the heat flux is known (and, thus, the choice of the number of future time steps is made easier), it is often not too difficult to adjust the number of future time steps to obtain good results. A good selection is obtained by observing the outcome of the estimation (paying attention to regions of rapid change) and the residuals. The use of the residual principle for selecting the number of future time steps is discussed by Beck.[18]

Summary and Conclusions

The analysis of experiments to determine the thermal properties of a carbon-carbon composite material was presented. The composite material was made by Carbon-Carbon Advanced Technologies, Fort Worth, Texas, and is called CC1 2-D composite made from fiberite K-641 fully densified. Thermal properties are calculated for the carbon-carbon composite from one-dimensional experiments using measured temperature and heat flux histories. The reverse problem of estimating the surface heat flux using the previously determined thermal properties and temperature measurements was also shown for the one-dimensional case.

The methods were very powerful; thermal conductivity $\kappa_{y,cc}$, and volumetric heat capacity ρC_{cc} were determined simultaneously for temperatures up to 600°C from one-dimensional experiments. The models determined to represent the temperature dependence of the properties were

$$\kappa_{y,cc} = 3.195 + 0.00756T - 0.813E\text{-}05T^2 \tag{8}$$

$$(\rho C)_{cc} = (1.280 + 0.00526T - 0.363E\text{-}05T^2) \times 10^6 \tag{9}$$

The thermal properties are within 2% for $\kappa_{y,cc}$ and 6% for $(\rho C)_{cc}$.

Recovering the surface heat flux (IHCP) using the estimated thermal properties was very accurate for the one-dimensional experiments.

Acknowledgments

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